Physical Accuracy and Modeling Robustness of Motional Impedance Models

by

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History and Motivation: 1930s

- The basic electroacoustic model for *direct radiator loudspeakers* was developed in the 1930s
- From Olson's *Elements of Acoustical Engineering* (1940):

$$z_{EM} = \frac{(Bl)^2}{z_{MT}}$$

where B = flux density in the air gap, in gausses,
l = length of the conductor, in centimeters, and
z_{MT} = total mechanical impedance, in mechanical ohms.

$$z_{MT} = r_M + j\omega m + \frac{1}{j\omega C_M}$$

where r_M = mechanical resistance, in mechanical ohms,
m = mass of the air load, cone and coil, in grams, and
C_M = compliance of the suspension system, in centimeters per dyne.

History and Motivation: 1970s to 1990s

- In the 1970s, it was recognized that compliance was not static but exhibited **frequency-dependent** viscoelastic behaviour (Elliott, JAES 26 (1978) 1001).

COMPLIANCE - THE PROBLEM PARAMETER.

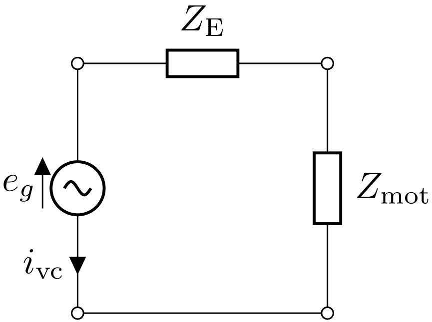
Because elastomers are used in the suspension system, the compliance term: is non-linear with displacement, has frequency-dependent dynamic values at very low frequencies, has a larger static value than dynamic value, suffers from hysteresis and gives rise to a frequency-dependent loss component. Some of these characteristics are illustrated in

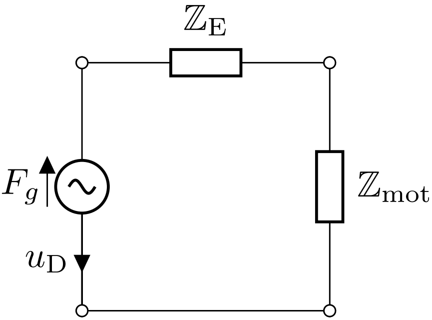
- In the 1990s, the first empirical creep-compliance models were explored (Knudsen and Jensen, JAES 41 (1993) 3).

Present Status of Creep-Compliance Models

- At present, there are a handful of established **creep-compliance** models in use:
  1. 1993: Knudsen (**LOG**)
  2. 2010: Ritter creep (**3PC**)
  3. 2011: Thorborg *f*-dependent damping (**FDD, SI-LOG**)
  4. 2016: Novak fractional derivative (**FD**)
- These models replace 1-parameter static compliance with a 2 or 3-parameter form.

Electrical and Mechanical Circuits for Transducer





Electrical circuit

$V \to e_g$

$I \to i_\mathrm{vc}$

$R \to Z = (B\ell)^2/\mathbb{Z}$

Mechanical circuit

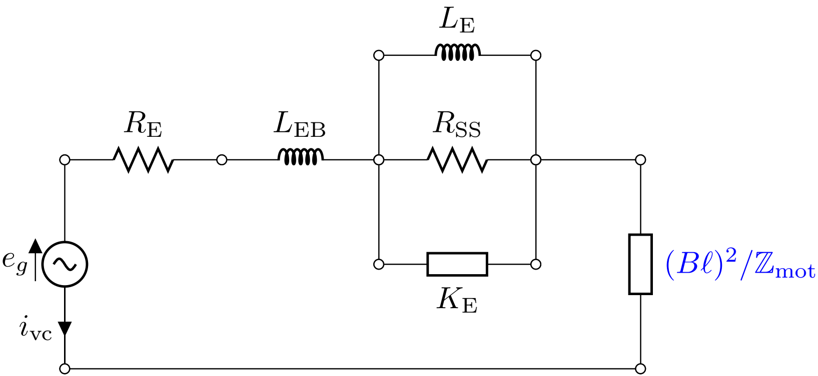
$V \to F_g = e_g(B\ell)/Z_\mathrm{E}$

$I \to u_\mathrm{D}$

$R \to \mathbb{Z} = (B\ell)^2/Z$

Complete Electrical Circuit for Transducer

Z_E from Thorborg and Futtrup, JAES 59 (2011) 612.



$$Z = Z_E + \frac{(B\ell)^2}{\mathbb{Z}_{mot}}$$

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Traditional Static Compliance (TS)

$$\mathbb{Z}_{\mathrm{mot}} = i\omega M_{\mathrm{MS}} + R_{\mathrm{MS}} + \frac{1}{i\omega C_{\mathrm{MS}}}$$

- Basis of technical datasheets
- C_{MS} is the **fixed compliance**
- A textbook damped harmonic oscillator
  $\longrightarrow$ $k = 1/C_{\mathrm{MS}}$ the **spring constant**

Knudsen LOG Model

$$\mathbb{Z}_{\text{mot}} = i\omega M_{\text{MS}} + R_{\text{MS}} + \frac{1}{i\omega C_0 \left[1 - \beta \ln(i\omega)\right]}$$

- Two compliance parameters: (C_0, β)
- Knudsen and Jensen, JAES 41 (1993) 3
- Simple but **very accurate** for typical drivers
- Resistance and compliance now **depend on frequency**

Ritter 3-parameter Creep (3PC)

$$\mathbb{Z}_{\text{mot}} = i\omega M_{\text{MS}} + R_{\text{MS}} + \frac{1}{i\omega C_0 \left[1 - \beta \ln\left(\frac{i\omega}{\omega_0 + i\omega}\right)\right]}$$

- Three compliance parameters: (C_0, β, ω_0)
- Ritter and Agerkvist, JAES 129, paper 8217 (2010)
- High-frequency cutoff to LOG model for $\omega \gg \omega_0$
- $\omega_0 = 1/\tau_{\text{min}} = 2\pi f_{\text{crit}}$

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Thorborg-Futtrup SI-LOG and FDD Models

$$\mathbb{Z}_{\text{mot}} = i\omega M_{\text{MS}} + R_{\text{MS}} + \frac{1}{i\omega C_0}\left(\frac{1 + i\Lambda}{1 - \beta \ln \omega}\right)$$

- Three compliance parameters: (C_0, Λ, β)
- Thorborg and Futtrup, JAES 59 (2011) 612
- More general form of storage versus loss compliance
- Used on ScanSpeak datasheets: **FDD** $\rightarrow \beta = 0$

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Novak Fractional Derivative (FD) Model

$$\mathbb{Z}_{\text{mot}} = i\omega M_{\text{MS}} + R_{\text{MS}} + \frac{1 + \eta (i\omega)^{\beta}}{i\omega C_0}$$

- Three compliance parameters: (C_0, η, β)
- Novak, JAES 64 (2016) 35
- Clever alternative to LOG-type models

$$\left(\frac{\partial}{\partial t}\right)^{\beta} e^{st} \doteq s^{\beta} e^{st}$$

Drivers tested

Name	D (cm)	Damping	VC Former	Copper
FU	10	medium-low	alum	cap
L16	15	medium-low	alum	ring below gap
W18	18	medium-low	alum	rings above/ below gap
L19	18	ultra-low	glass-fiber	rings above/ below gap
W26	26	ultra-low	kapton	ring below gap

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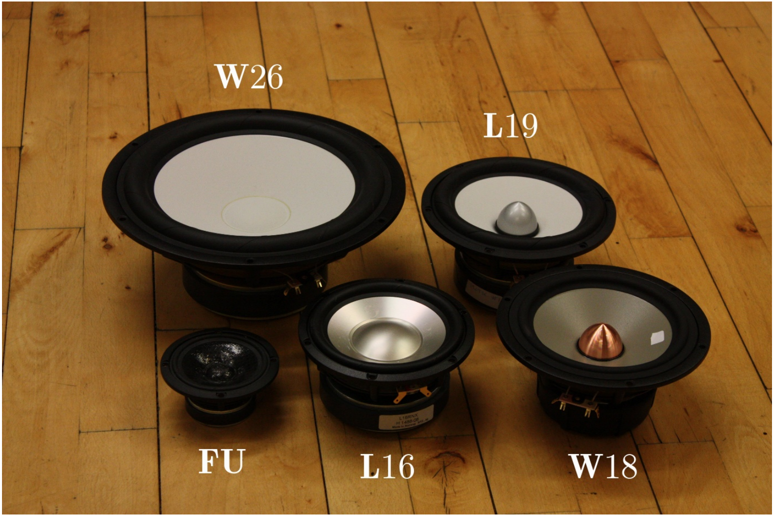
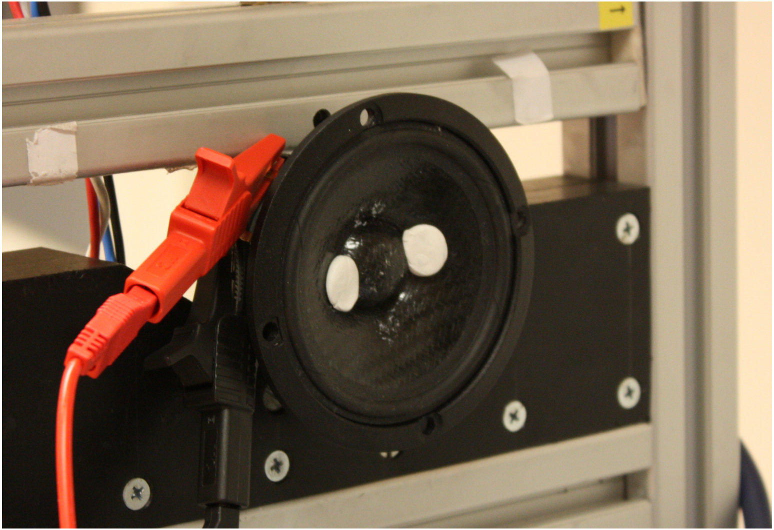
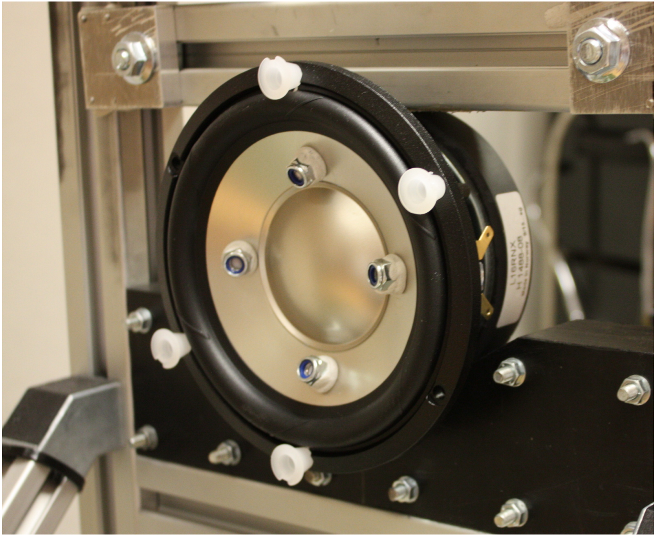
5 drivers



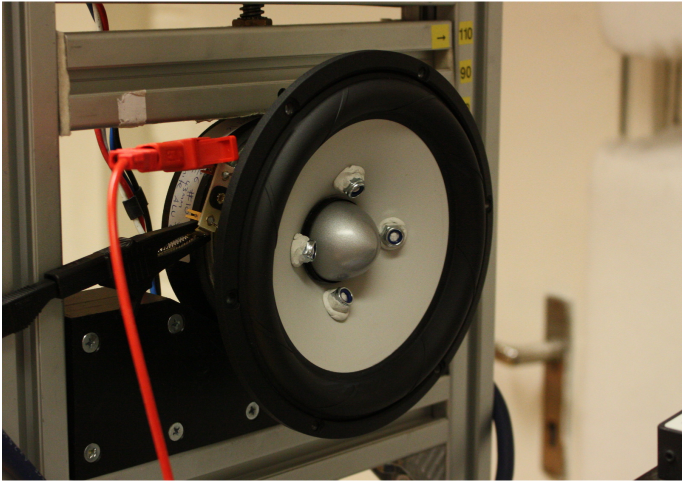
FU10



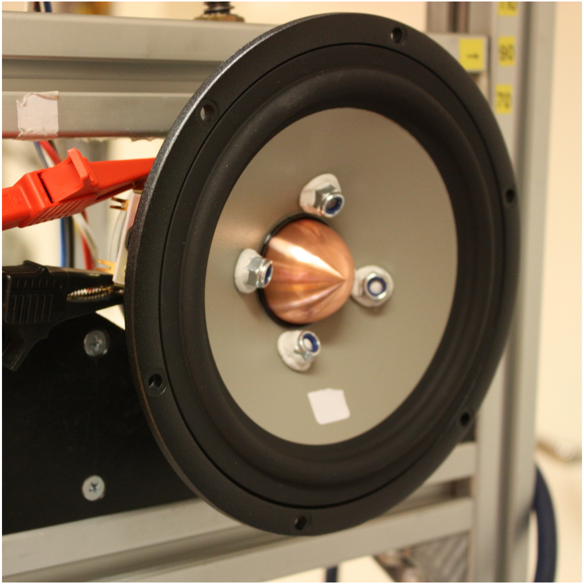
L16



L19



W18



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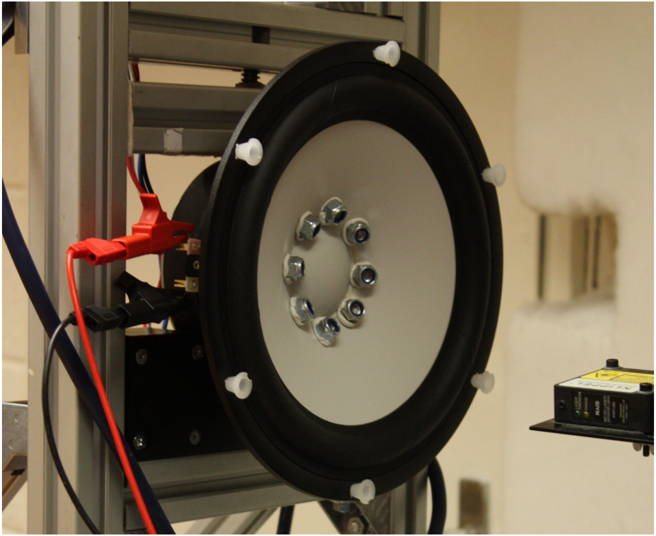
W26



Accurate Added-Mass Determination is Critical

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Electrical Measurement System

- Smith & Larson **Woofer Tester Pro**
- **Continuous-sine** measurement (approx 400 points)
- Constant voltage (242 mV) method

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$$Z(\omega) = \overbrace{Z_{\mathrm{E}}(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega M_{\mathrm{MS}} + f(\omega)}}^{\text{Motional Impedance}}$$

- $f(\omega)$ is model dependent
- Assume all mass dependence captured by M_{MS}
- Neglect nonlinear effects, so need to use low voltage

$$Z^{(0)}(\omega) = \overbrace{Z_E(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega M_{MS} + f(\omega)}}^{\text{Motional Impedance}}$$

1. Perform 3 measurements:
   - Cone unweighted: $Z^{(0)}$
   - Cone with added mass m_1 attached: $Z^{(1)}$
   - Cone with added mass m_2 attached: $Z^{(2)}$

Measurement and Analysis Workflow
Added mass

$$Z^{(1)}(\omega) = \overbrace{Z_E(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega(M_{MS} + m_1) + f(\omega)}}^{\text{Motional Impedance}}$$

1. Perform 3 measurements:
   - Cone unweighted: $Z^{(0)}$
   - Cone with added mass m_1 attached: $Z^{(1)}$
   - Cone with added mass m_2 attached: $Z^{(2)}$

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$$Z^{(2)}(\omega) = \overbrace{Z_E(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega(M_{MS} + m_2) + f(\omega)}}^{\text{Motional Impedance}}$$

1. Perform 3 measurements:
   - Cone unweighted: $Z^{(0)}$
   - Cone with added mass m_1 attached: $Z^{(1)}$
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Measurement and Analysis Workflow
Extract pure motional impedance

$$Z(\omega) = \overbrace{Z_{\mathrm{E}}(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega M_{\mathrm{MS}} + f(\omega)}}^{\text{Motional Impedance}}$$

❷ Subtract to **remove electrical impedance** from data

$$\Delta Z_1 \doteq Z^{(0)} - Z^{(1)} \quad \text{and} \quad \Delta Z_2 \doteq Z^{(0)} - Z^{(2)}$$

and compute model-free motional impedance

$$Z^{*}_{\mathrm{mot}} \doteq \frac{(1-\mu)\Delta Z_1 \Delta Z_2}{\Delta Z_2 - \mu \Delta Z_1}$$

where $\mu = m_2/m_1$.

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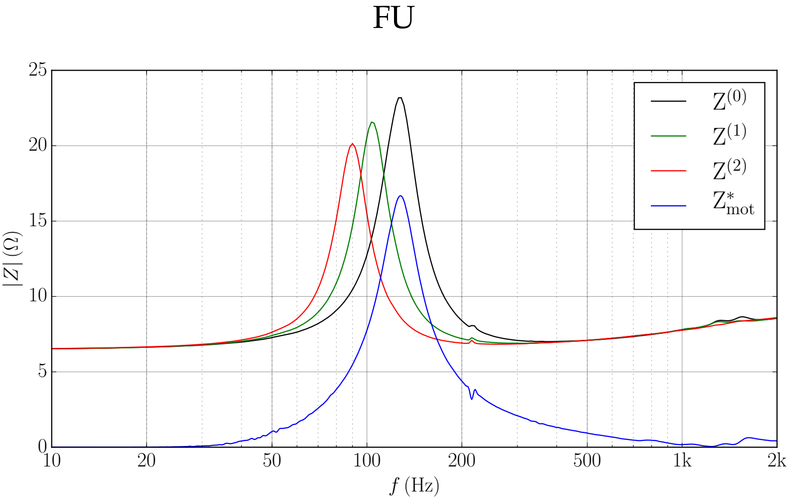
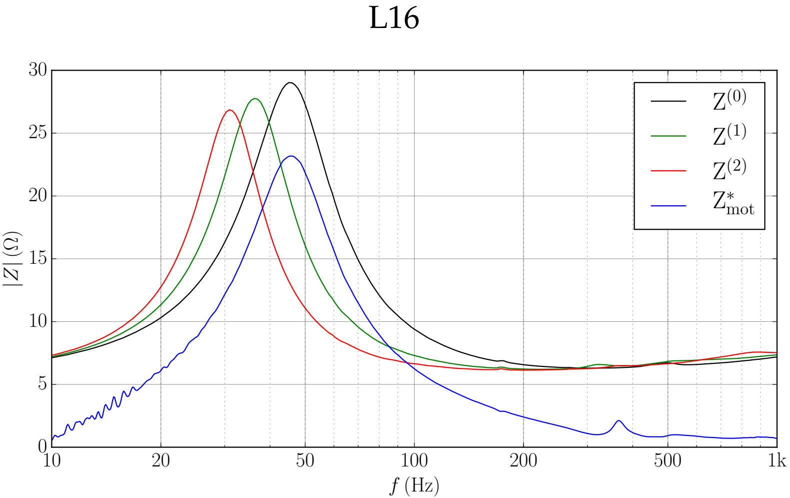
Example Z^*_{mot} curves



Example Z^*_{mot} curves



Measurement and Analysis Workflow

Determine $B\ell$

$$Z(\omega) = \overbrace{Z_{\mathrm{E}}(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega M_{\mathrm{MS}} + f(\omega)}}^{\text{Motional Impedance}}$$

3. Compute $B\ell$ using **frequency-average**

$$(B\ell)^2 = m_1 \left\langle \frac{i\omega Z^*_{\mathrm{mot}}(Z^*_{\mathrm{mot}} - \Delta Z_1)}{\Delta Z_1} \right\rangle_{\omega_1}^{\omega_2}$$

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Measurement and Analysis Workflow

Motional impedance fit

$$Z(\omega) = \overbrace{Z_E(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega M_{MS} + f(\omega)}}^{\text{Motional Impedance}}$$

❹ Fit $\mathbb{Z}_{mot}$ using complex least-squares method

$$\mathbb{Z}_{mot}^{fit} : \quad i\omega M_{MS} + R_{MS} + \cdots = \frac{(B\ell)^2}{Z_{mot}^*}$$

$$Z(\omega) = \overbrace{Z_{\mathrm{E}}(\omega)}^{\text{Electrical Impedance}} + \overbrace{\frac{(B\ell)^2}{i\omega M_{\mathrm{MS}} + f(\omega)}}^{\text{Motional Impedance}}$$

4. Fit Z_{E} using complex least squares method

$$Z_{\mathrm{E}}^{\mathrm{fit}}: \quad R_{\mathrm{E}} + i\omega L_{\mathrm{EB}} + \cdots = Z^{(0)}(\omega) - \frac{(B\ell)^2}{\mathbb{Z}_{\mathrm{mot}}^{\mathrm{FIT}}(\omega)}$$

Illustration of Fit Regions

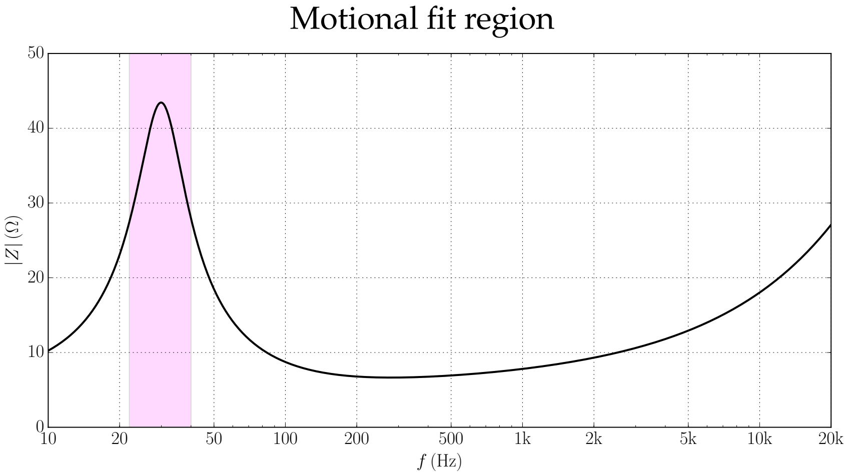


Illustration of Fit Regions

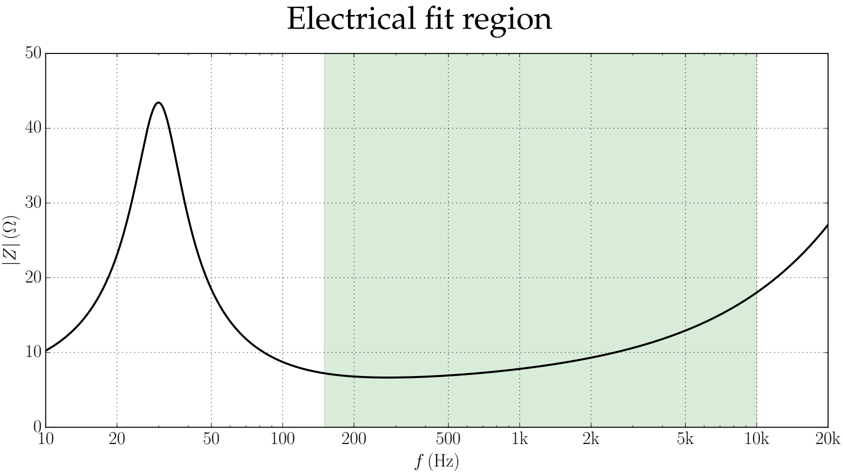
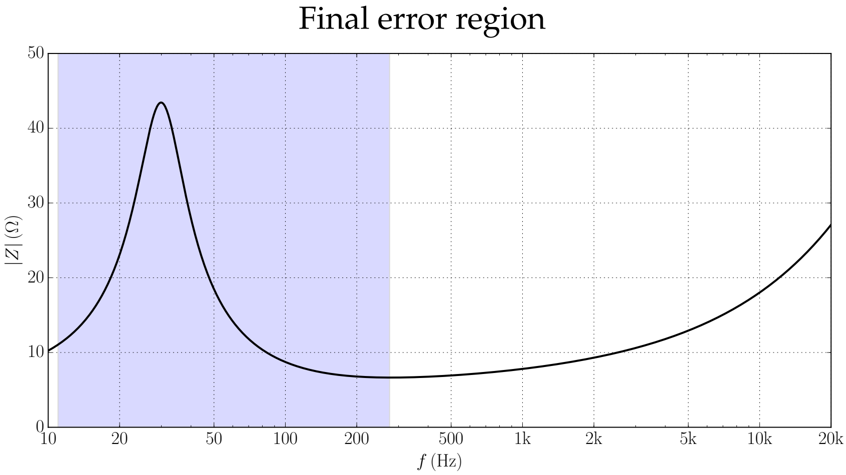
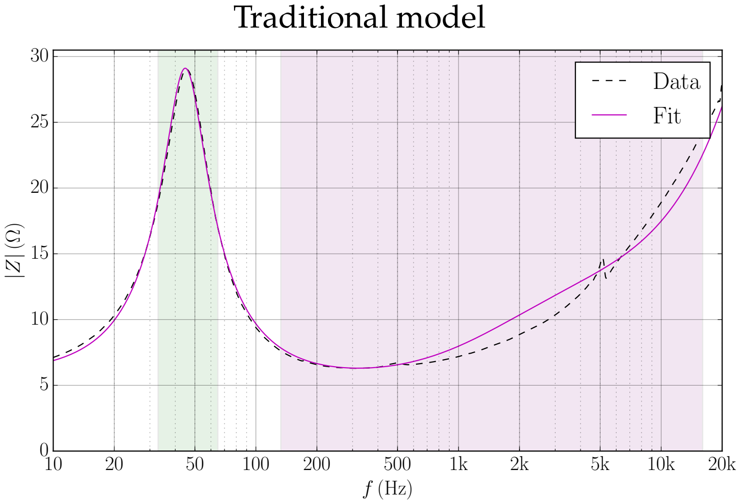


Illustration of Fit Regions

Other regions are adjusted to minimize total error here



Fit Example: L16
Impedance



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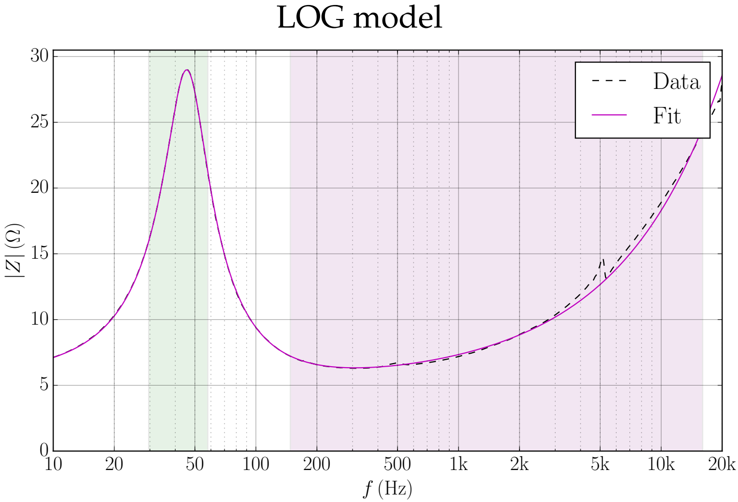
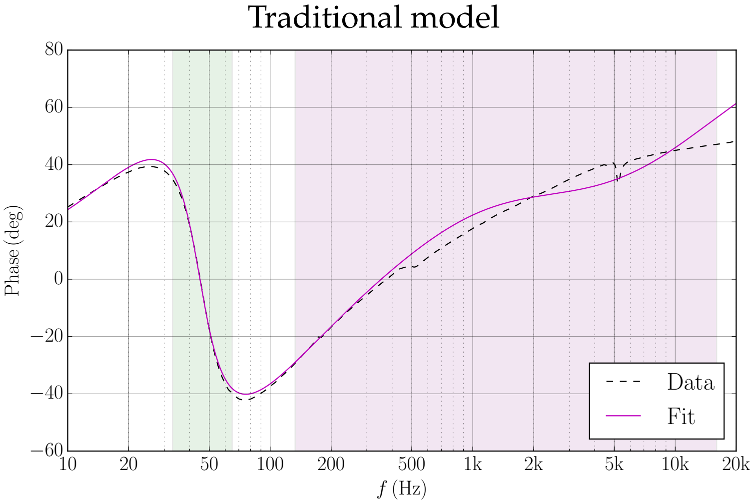
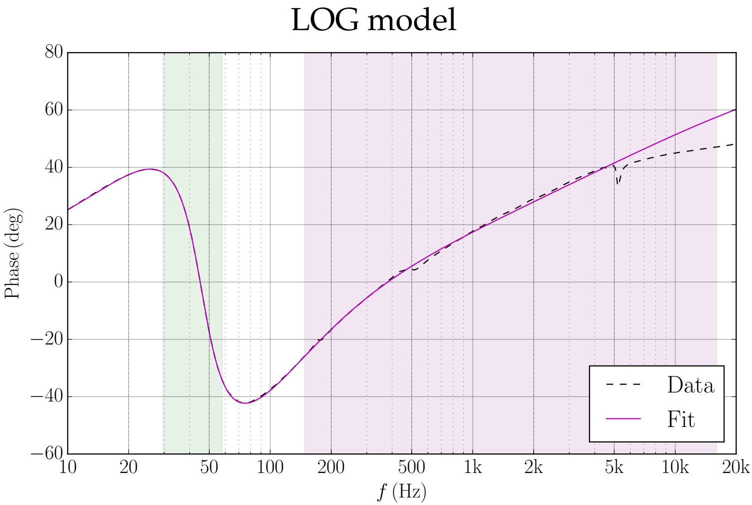
Fit Example: L16
Impedance



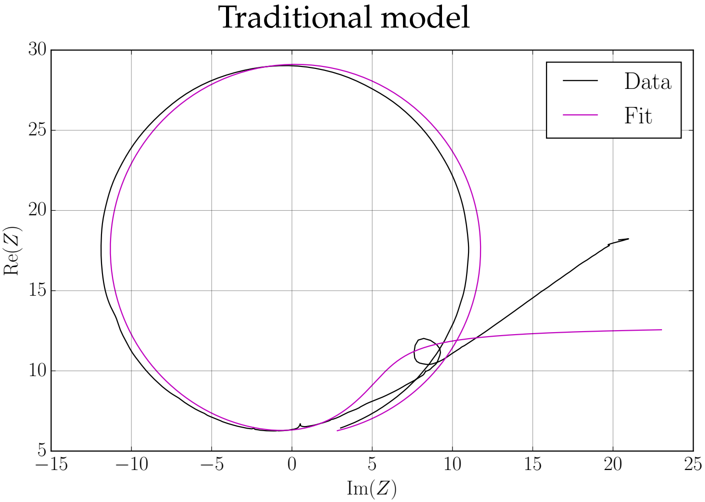
Fit Example: L16
Phase



Fit Example: L16
Phase

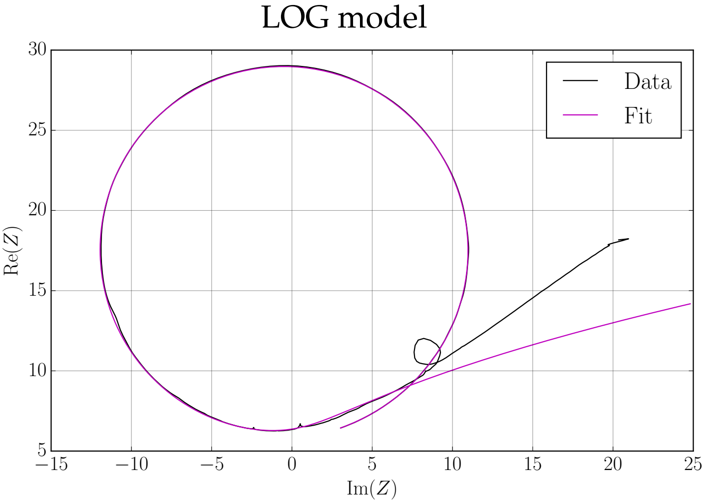


Fit Example: L16
Nyquist plot



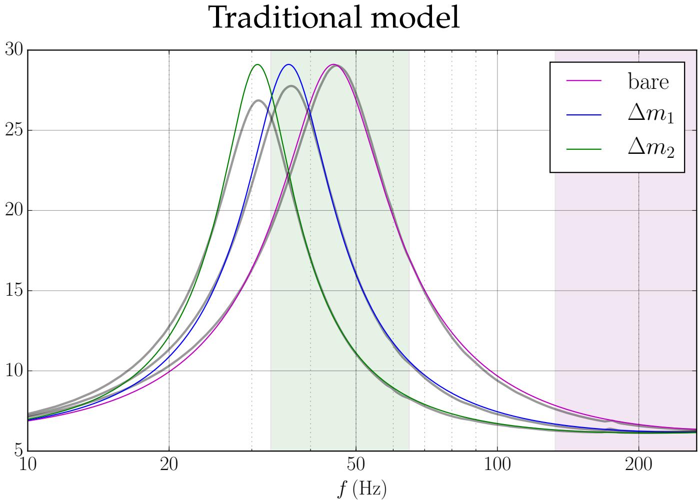
Fit Example: L16

Nyquist plot



Fit Example: L16

Z comparison



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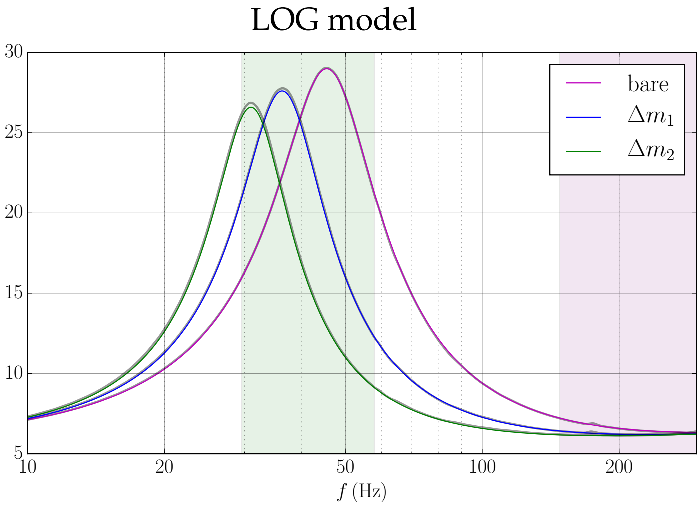
Fit Example: L16
Z comparison



Fit Example: L16

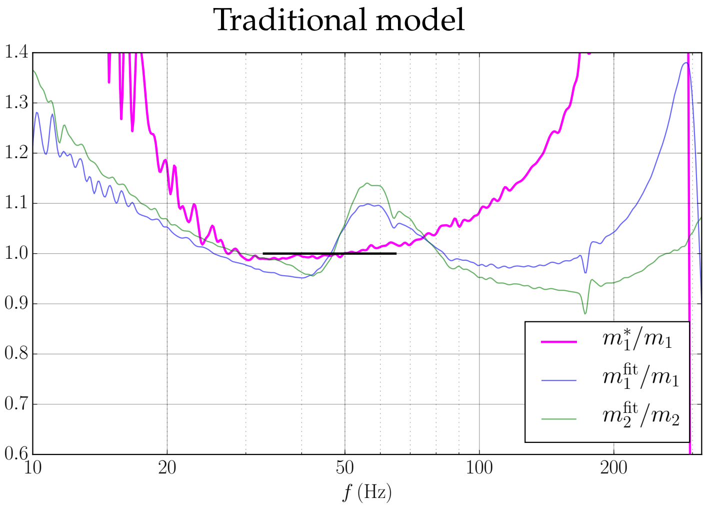
Mass consistency formulae

$$m_1^* = \frac{(B\ell)^2}{i\omega} \frac{\Delta Z_1}{Z_{\text{mot}}^* (Z_{\text{mot}}^* - \Delta Z_1)}$$

$$m_1^{\text{fit}} = \frac{(B\ell)^2}{i\omega} \frac{\Delta Z_1}{Z_{\text{mot}}^{\text{fit}} (Z_{\text{mot}}^{\text{fit}} - \Delta Z_1)}$$

$$m_2^{\text{fit}} = \frac{(B\ell)^2}{i\omega} \frac{\Delta Z_2}{Z_{\text{mot}}^{\text{fit}} (Z_{\text{mot}}^{\text{fit}} - \Delta Z_2)}$$

Fit Example: L16
Mass consistency



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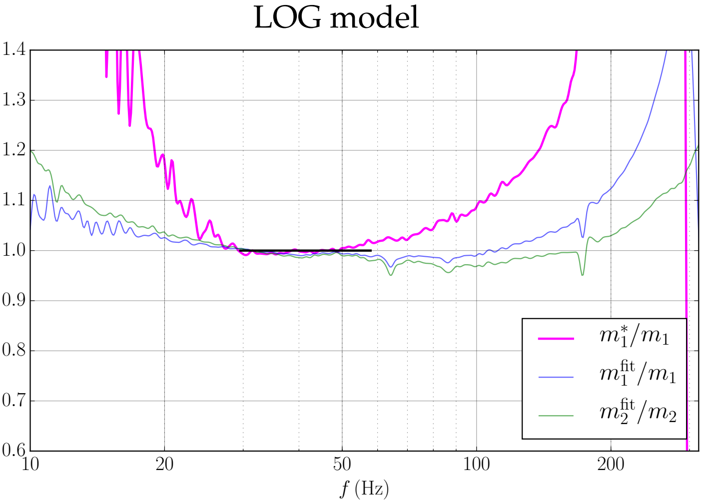
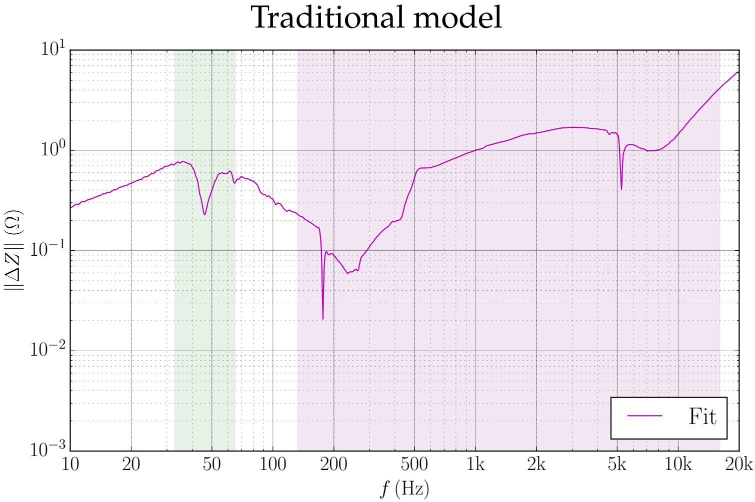
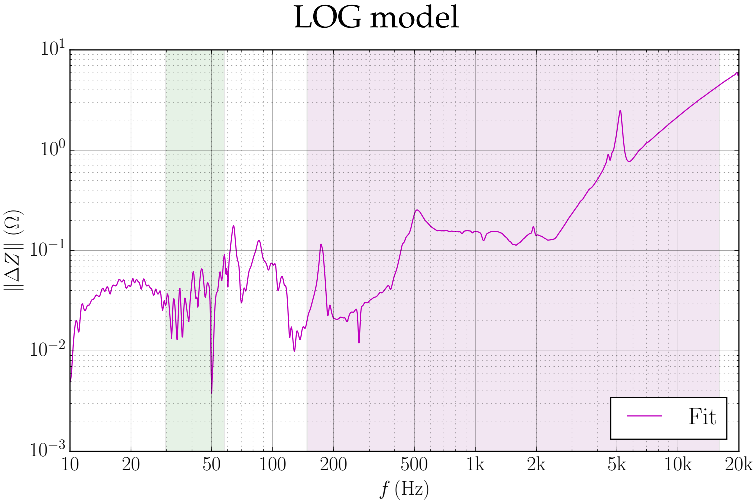
Fit Example: L16
Mass consistency



Fit Example: L16
Fit error



Fit Example: L16
Fit error



Driver-Model Comparison Matrix

Average fit error in Ohms

	TS	FDD	LOG	SI-LOG	3PC	FD
FU	0.089	0.025	0.026	0.016	0.026	0.025
L16	0.170	0.074	0.019	0.013	0.018	0.020
W18	0.160	0.047	0.009	0.009	0.010	0.008
L19	0.342	0.135	0.079	0.081	0.026	0.196
W26	0.216	0.046	0.033	0.031	0.032	0.032

Conclusions
Comments on model robustness and accuracy

- 2-parameter **LOG model** gives excellent balance of **simplicity** versus **accuracy**
- SI-LOG and FD models may be slightly more accurate in **some cases**
- 3PC model may be the **most robust** (more testing required)
- All models yield **frequency-dependent damping** absent from traditional model
- **Added mass** measurements require care and precision
- **Electrical measurement system** should have **high S/N**

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